Twisted Family Structure and Neutrino Large Mixing

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Abstract

I demonstrate that neutrino large mixing between ν_μ and ν_τ are naturally reproduced using a novel mechanism called 'E-twisting'in a supersymmetric E_6 grand unification model. This model explains all the characteristic features of the quark/lepton Dirac masses as well as the neutrino's Majorana masses despite the fact that all the members in **27** of each generation are assigned a common family charge. Most remarkably, this model yields a novel relation which gives the 2-3 lepton mixing angle $\theta_{\mu\tau}$ in terms of quark masses and CKM mixing: $\tan\theta_{\mu\tau} = (m_b/m_s)V_{cb}$, which is a kind of $SO(10)$ GUT relation similar to the celebrated $SU(5)$ bottom-tau mass ratio. This relation is a result of a common 'twisted $SO(10)$' structure [1].

A remarkable fact observed in SuperKamiokande [1] is the very large lepton mixing, which is in a sharp contrast to the quark sector where the CKM mixings are all small. Why can such a large difference occurs between the quark and lepton sectors? This is a challenging question for any particle physicist who tries to find grand unified theories (GUTs). Clearly any GUT which treats the three families of quarks and leptons as a mere repetition no longer works. We need some new mechanism of family structure. Lots of proposals have been made on the origin of this large lepton mixing angles [2]. On the other hand, the SK results indicate larger unification groups than $SU(5)$ including left-right symmetry, in which large neutrino mixings seem unnatural, since in such larger GUT groups, for example, $SO(10)$ GUT, all the fermions of a family are combined into a single representation, and the most natural prediction would be that the neutrino mixing is also very small with hierarchical masses.

Recently we have constructed a supersymmetric E_6 unified model with an extra $U(1)$ family charge[3]. There we have shown that E-twisted family structure can reproduce all the characteristic features of the fermion masses, not only the quark/lepton Dirac masses but also the neutrino Majorana masses. Despite the fact that a common $U(1)$ charge is assigned to all the members in a fundamental representation **27** of E_6 for each family, the model well explains all the qualitative feature of different mass hierarchies among families and between up and down quark sectors, as well as the mixing angles. In this scenario we have found that in the framework of a supersymmetric E_6 grand unified model [3], the twisted family

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structure yields a novel relation

$$\tan\theta_{\mu\tau} \;=\; \frac{m_b}{m_s}V_{cb}, \tag{1}$$

which I shall compare with the experiments later. Leaving the details in the papers [3], I here explain the essence of the model. In the supersymmetric E_6 model, in addition to E_6 gauge vector multiplet, we introduce chiral matter multiplets corresponding to the three families, (Ψ_i ($i=1,2,3$)) and a pair of Higgs fields, which is introduced mainly for the electroweak symmetry breaking,(H, $\bar{H}$) [2]. In table 1, we summarize all the fields we need in our model. The E_6 singlet field Θ with $U(1)$ charge -1 plays an important role that its suitable powers

	Ψ_1	Ψ_2	Ψ_3	H	$\bar{H}$	Φ	$\bar{\Phi}$	ϕ	Θ
E_6	$\mathbf{27}$	$\mathbf{27}$	$\mathbf{27}$	$\mathbf{27}$	$\mathbf{27}^*$	$\mathbf{27}$	$\mathbf{27}^*$	$\mathbf{78}$	$\mathbf{1}$
$U(1)$ charge	3	2	0	0	0	-4	4	-2	-1
R parity	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$+$

Table 1: E_6 representations and $U(1)$ charge assignment.

compensate the mismatch of the $U(1)$ charge in the superpotential interaction terms. The $U(1)$ flavor symmetry discriminates families and induces hierarchy between them. Note that all the quarks and leptons in one generation have a common $U(1)$ quantum number.

The following Yukawa superpotentials which are invariant under R parity, $U(1)$ and E_6 will give masses of matter superfields $\Psi_i(\mathbf{27})$ [3],

$$W_Y(H) = y_{ij}\,\Psi_i(\mathbf{27})\Psi_j(\mathbf{27})H(\mathbf{27})\left(\frac{\Theta}{M_P}\right)^{f_i+f_j}, \tag{2}$$

where f_i denotes the $U(1)$ charge of i-th family. With the coupling constants y of order 1, the effective Yukawa coupling constants are associated with additional powers of $\lambda = \langle\Theta\rangle/M_P$[5], which we assume is of the order of the Cabibbo angle $\lambda\sim 0.22$. We also suppose that only the $SU(2)$ doublet components of H can have the electroweak scale vacuum expectation value (VEV).

An interesting fact is that there are two $\mathbf{5}^*$'s of $SU(5)$ in each $\mathbf{27}$, i.e., $\mathbf{5}^*$ of $\mathbf{16}$ of $SO(10)$ (($\mathbf{16},\mathbf{5}^*$)) and $\mathbf{5}^*$ of $\mathbf{10}$ (($\mathbf{10},\mathbf{5}^*$)). Those may be called 'E-parity' doublet. It is this doubling that we have a freedom to choose the low-energy $\mathbf{5}^*$ candidates. This actually implies that the embedding of $SO(10)$ into E_6 such that $SU(5)_{\rm GG}\subset SO(10)\subset E_6$ with Georgi-Glashow $SU(5)_{\rm GG}$, possesses a freedom of rotation of $SU(2)_R$. The doubling of $\mathbf{5}^*$'s in each $\mathbf{27}$ also provides the low-energy surviving down-type Higgs field with the freedom of mixing parameter between two $\mathbf{5}^*$'s in $H(\mathbf{27})$:

$$H(\mathbf{5}^*) \;=\; H(\mathbf{10},\mathbf{5}^*)\cos\theta + H(\mathbf{16},\mathbf{5}^*)\sin\theta. \tag{3}$$

[2]In order to give all the unwanted fermions to get heavy masses. we need another Higgs pair, (Φ, $\bar{\Phi}$) and which are responsible for realizing the E-twisted family structure. Also we have to add a chiral Higgs multiplet $\phi(\mathbf{78})$ in order to break the GUT to the standard gauge group. Here we neglect those fields and start with the low energy fermions realized in twisted family structure.

[3] There are other superpotentials including the Higgs fields, $\phi(\mathbf{78})$ and $\Phi(\mathbf{27})$, whose family charges are not zero and will contribute to the Yukawa terms of the 2nd and 1st families.

Now we pick up the low-energy matter fields among the three $\Psi_i(\mathbf{27})$ of the above. The up-quark sector is unique since $\mathbf{10}$ and $\mathbf{5}$ of $SU(5)$ appear only once in each $\mathbf{27}$. As for the down quarks, there is a freedom for choosing three from six $\mathbf{5}^*$s in three $\Psi_i(\mathbf{27})$s. We have classified possible typical scenarios in Ref. [3]; (i) Parallel family structure, (ii) Non-parallel family structure, and (iii) E-twisted structure. Among these three possibilities, we here take the simplest and most attractive option, namely the E-twisted family:

$$(\mathbf{5}^*_1,\ \mathbf{5}^*_2,\ \mathbf{5}^*_3) \ = \ (\Psi_1(\mathbf{16},\mathbf{5}^*),\ \Psi_2(\mathbf{16},\mathbf{5}^*),\ \Psi_3(\mathbf{10},\mathbf{5}^*)). \tag{4}$$

This structure implies that the third family $\mathbf{5}^*$ belongs to $\mathbf{10}$ of $SO(10)$. This twisting is realized by the suitable VEVs of the Higgs fields(the details will be found in Ref. [3]).

Let us here concentrate ourselves on the 2nd and 3rd families and see what happens to their masses and mixings. The 2×2 mass matirces for the up-quark, down-quark and charged-lepton, M_u, M_d and M_e, are expressed as,

$$M_u = \begin{array}{c} \\ u_2 \\ u_3 \end{array}\!\!\begin{array}{c} \begin{array}{cc} u_2^c & u_3^c \end{array} \\ \begin{pmatrix} y_{22} & y_{23} \\ y_{32} & y_{33} \end{pmatrix} \end{array} v\sin\beta = \begin{array}{c} \\ u_2 \\ u_3 \end{array}\!\!\begin{array}{c} \begin{array}{cc} u_2^c & u_3^c \end{array} \\ \begin{pmatrix} * & f\lambda^2 \\ * & 1 \end{pmatrix} \end{array} v y_{33}\sin\beta, \tag{5}$$

$$M_d = \begin{array}{c} \\ d_2 \\ d_3 \end{array}\!\!\begin{array}{c} \begin{array}{cc} d_2'^c & D_3^c \end{array} \\ \begin{pmatrix} z_{22}^d\cos\theta & y_{23}\sin\theta \\ z_{32}\cos\theta & y_{33}\sin\theta \end{pmatrix} \end{array} v\cos\beta = \begin{array}{c} \\ d_2 \\ d_3 \end{array}\!\!\begin{array}{c} \begin{array}{cc} d_2'^c & D_3^c \end{array} \\ \begin{pmatrix} e\lambda^2 & f\lambda^2 \\ h & 1 \end{pmatrix} \end{array} v y_{33}\sin\theta\cos\beta. \tag{6}$$

$$M_e^{\mathrm{T}} = \begin{array}{c} \\ e_2^c \\ e_3^c \end{array}\!\!\begin{array}{c} \begin{array}{cc} e_2' & E_3 \end{array} \\ \begin{pmatrix} z_{22}^e\cos\theta & y_{23}^e\sin\theta \\ z_{32}q\cos\theta & y_{33}\sin\theta \end{pmatrix} \end{array} v\cos\beta = \begin{array}{c} \\ e_2^c \\ e_3^c \end{array}\!\!\begin{array}{c} \begin{array}{cc} e_2' & E_3 \end{array} \\ \begin{pmatrix} * & * \\ h & 1 \end{pmatrix} \end{array} v y_{33}\sin\theta\cos\beta. \tag{7}$$

where $\tan\beta$ is the mixing angle of two light Higgs doublets and v is the VEV of the standard Higgs field $H(\mathbf{27})$. We rewrite by using simple notations in the third terms with $*$ being irrelevant for our present discussions [4]. Note that by taking the mixing $\sin\theta$ of the Higgs field $H(\mathbf{27})$ of Eq.(3), to be of order λ^2, h becomes of order 1. It is easy to obtain h from the bottom and strange quark masses and mixing angle, m_b, m_s, V_{cb} from Eqs.(5) and ((6)). Noting that h gives directly the lepton mixing angle $\tan\theta_{\mu\tau} = h$ [5], we can get the novel relation Eq.(1), or equivalently,

$$\sin^2 2\theta_{\mu\tau} \ = \ \frac{4V_{cb}^2\left(\dfrac{m_s}{m_b}\right)^2}{\left[V_{cb}^2+\left(\dfrac{m_s}{m_b}\right)^2\right]^2}\,. \tag{8}$$

By taking the experimental value of $x = V_{cb}m_b/m_S$, $1 \leq x \leq 1.68$, we can calculate the left hand side of Eq.(8), namely,

$$0.78 \leq \sin^2 2\theta_{\mu\tau} \leq 1. \tag{9}$$

[4] We have assumed that the main contribution comes only from the Higgs field $H(\mathbf{27})$ at least for the quark mass matrix of 33 and 23 elements in the quark mass matrices, M_u and M_d.

[5] We can confirm that the right handed Majorana mass term indicates very small mixing and neutrino mixing mainly comes from lepton mixing.

which is remarkably in good agreement with the large lepton mixing recently observed[1].

This relation can be obtained from more general framework using a kind of $SO(10)$ GUT and may be called the second q-l relation, similar to the first q-l relation, i.e., the celebrated bottom-tau mass ratio of $SU(5)$[4]. It is interesting that this relation can be most easily obtained from the twisted E_6) model. We would like to remark that our E_6 twisted model can also explain why the bottom quark mass is smaller by almost λ^2 than that of the top quark. This also comes from the Higgs mixing factor $\sin\theta$ in M_d.

To conclude, we have found that the twisted E_6 model can explain the up-down mystery ($m_t \gg m_b$), as well as the down-lepton mystery ($\theta_{\mu\tau} \gg \theta_{cb}$). It is well known that E_6 gauge symmetry is naturally obtained from the 10 dimensional $E_8 \times E_8$ heterotic string theory by the Calabi-Yau compactification into 4 dimensions. Our results are interesting and encouraging and may open the door for finding out more fundamental stringy GUTs including gravity.

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